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Directivity analysis and modeling simulation of laser altimeter measurement

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Abstract. Precision tracking turntable is the key unit of high-precision laser altimeter. Its accuracy directly affects the overall accuracy of laser altimeter measurement, and how to ensure the authenticity of the height measurement in the laser altimeter measurement process is the most important technical problem in the design of laser altimeter. Based on the "directivity" model of tracking technology, this paper discusses the geocentric directivity of the laser altimeter. According to this method, the geocentric directivity model is constructed, and the mathematical expression of the geocentric directional adjustment quantity is deduced. The simulation with Matlab results show that the tracking trajectory is basically coincident with the original trajectory, and the measurement error is 5cm. And the tracking error reaches 0.003° by debugging with the turntable. These results prove that the measurement accuracy can be improved by using this method in the measurement process.

Key words. Laser altimeter, geocentric directivity, matlab, error.

1. Introduction

For the past measurement of height and distance , the design of transmitter and receiver is usually discussed. Relatively speaking, the research on how to adjust the direction of measurement has not brought about the widespread attention. For plane ranging, if the measurement between two points, there will not be deviation; but if the measurement between point and surface, there will be angle deviation problem. Generally, in the height measurement, the slight deviation of the angle will make the measurement result in larger measurement error [1]. Therefore, how to ensure the directional accuracy of altimeter is the premise of altimeter measurement. Taking

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the altimeter measurement as an example, the geocentric directivity problem of the laser altimeter is analyzed.

Accurately pointing the laser altimeter to the center of the Earth is the ultimate goal of a laser altimeter stability attitude control system [2][3], it is a fixed reference frame, subsequently according to the direction of the laser altimeter measuring actual direction line vectors and theoretical direction (i.e. The direction of the earth's Center) between mathematical expressions of conversion adjust the amount of deduction.

2. The establishment of coordinate system

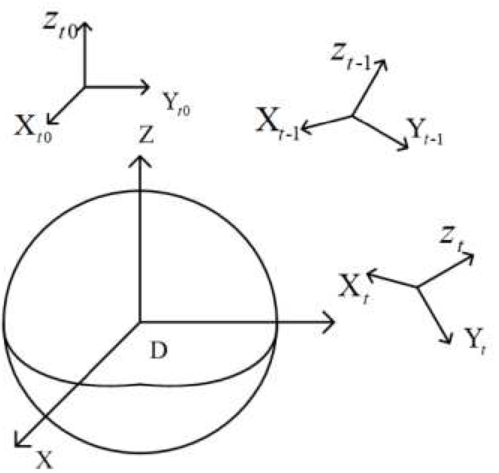

Fig. 1. Establishment of coordinate system

For the convenience of discussion the following coordinate system is established: the inertial coordinate system D of the earth is used as the reference coordinate system. two-axis adjustment PTZ mounted GPS and laser altimeter, the three to maintain a fixed connection. To coordinate the $\mathbf{Y}(X_tY_tZ_t)$ two-axis adjustment PTZ where the movement of objects (for) coordinate system, with the change of time t , the $\mathrm{Y}(X_tY_tZ_t)$ coordinates of the position and pitch angle changes and the origin of coordinates and two axis adjustment GPS keep the head, the pitch angle-$(\alpha, \beta, \gamma)$and pitch GPS and laser altimeter device the angle is consistent. As shown in Fig 1, the laser altimeter device in two-axis adjustment PTZ on the head, the origin of coordinates of laser device coordinates $\mathrm{A}(x, y, z)$ and two-axis adjustment PTZ is consistent with the coordinates obtained by the GPS, GPS access to the $\mathrm{A}(x, y, z)$ position of absolute coordinates on earth inertial coordinate system in coordinate system D.

3. Coordinate transformation

According to the first section, this section mainly describes the transformation between PTZ coordinate system and inertial coordinate system [4][5][6].

3.1. Initial measurement time t_0

From the first section to establish the coordinate system can get the first moment:
PTZ coordinate system$A_{t0}Y = (xY_{t0}, yY_{t0}, zY_{t0}, \alpha Y_{t0}, \beta Y_{t0}, \gamma Y_{t0})$
Inertial coordinate system$A_{t0}D = (xD_{t0}, yD_{t0}, zD_{t0}, \alpha D_{t0}, \beta D_{t0}, \gamma D_{t0})$
And according to $A_{t0}D = A_{t0}Y * BH_{t0-D}$ The BH_{t0-D} is the transformation matrix of the time t_0 PTZ coordinate system Y to the inertial coordinate system D.

At the initial time t_0, the PTZ coordinate system at this moment is marked as $X_{t_0}Y_{t_0}Z_{t_0}$. At this time, the position coordinates of the laser altimeter reading by GPS in the inertial coordinate system D is $(xD_{t_0}, yD_{t_0}, zD_{t_0})$, and the elevation angle of the laser altimeter device in the PTZ coordinate system $X_{t_0}Y_{t_0}Z_{t_0}$is $(\alpha Y_{t_0}, \beta Y_{t_0}, \gamma Y_{t_0})$.

$(\alpha Y_{t_0}, \beta Y_{t_0}, \gamma Y_{t_0})$*$BH_{Y_{t0}-D}$= $(\alpha D_{t_0}, \beta D_{t_0}, \gamma D_{t_0})$, Where $BH_{Y_{t0}-D}$ is the transformation matrix from the PTZ coordinate system to the earth's inertial coordinate system D at time t_0.

In the PTZ coordinates$X_{t_0}Y_{t_0}Z_{t_0}$, the laser altimeter position coordinates expressed as: $(0, 0, 0)$; Pitch angle coordinates: angle of gyroscope measurement : $(\alpha Y_{t_0}, \beta Y_{t_0}, \gamma Y_{t_0}) = (0, 0, 0)$.

In the earth inertial coordinate system, the laser altimeter device theory should point to the direction vector:

$$LDn_{t0} = (xD_{t_0}, yD_{t_0}, zD_{t_0}) - (0, 0, 0) = (xD_{t_0}, yD_{t_0}, zD_{t_0}) \quad (1)$$

After the calibration in the earth inertial coordinate system, the direction of the laser altimeter device is actually pointing Vector as:

$$SDn_{t0} = (xD_{t_0}, yD_{t_0}, zD_{t_0}) - (0, 0, 0) = (xD_{t_0}, yD_{t_0}, zD_{t_0}) \quad (2)$$

Therefore, the theoretical orientation of the laser altimeter device coincide with the actual orientation by the formulas (1) and (2)available at the initial time after calibration.

3.2. Measurement at any time

PTZ coordinate system: $A_tY = (xY_t, yY_t, zY_t, \alpha Y_t, \beta Y_t, \gamma Y_t)$
Inertial coordinate system: $A_tD = (xD_t, yD_t, zD_t, \alpha D_t, \beta D_t, \gamma D_t)$
And $A_tD = A_tY * BH_{t-D}$TheBH_{t-D} is the transformation matrix of the time tPTZ coordinate system Y to the inertial coordinate system D.

Adjustment in inertial coordinate system:
$\delta = (\alpha D_t, \beta D_t, \gamma D_t) - (\alpha D_{t-1}, \beta D_{t-1}, \gamma D_{t-1})$
Adjustment amount in PTZ coordinate system:$\xi = \delta * BH_{D-(t-1)}$Among them,

$BH_{D-(t-1)}$is the transformation matrix of the moment$t-1$ inertial coordinate system to the PTZ coordinate system.

At any time t, the laser altimeter is located at a certain point in the space, and its position coordinates in the earth inertial coordinate system is obtained by GPS, which is denoted as (xD_t, yD_t, zD_t). The pitch angle of the laser altimeter device in the PTZ coordinate system $X_tY_tZ_t$ can be measured as $(\alpha Y_t, \beta Y_t, \gamma Y_t)$ by the gyroscope.

The position coordinate of the laser altimeter in inertial coordinate system D is (xD_t, yD_t, zD_t); The pitch angle coordinates: $(\alpha D_{t_0}, \beta D_{t_0}, \gamma D_{t_0})^{*} BH_{t_0-t}$. The $BH_{t_0-t}{=}R_x(\alpha)_t * R_y(\beta)_t * R_z(\gamma)_t$ is the rotation matrix caused by pitch and yaw of the flight device.

In the PTZ coordinates $X_tY_tZ_t$, the laser altimeter position coordinates expressed as: $(0,0,0)$; Pitch angle coordinates: angle of gyroscope measurement : $(\alpha Y_t, \beta Y_t, \gamma Y_t)$.

In the earth inertial coordinate system$D$, the direction vector that the laser altimeter device theory should be pointed to is:

$$LDn_t = (xD_t, yD_t, zD_t) - (0,0,0) = (xD_t, yD_t, zD_t) \tag{3}$$

In the earth inertial coordinate system D, the direction vector of the laser altimeter device is actually the direction vector of the laser altimeter:

$$SDn_t = Sn_{t0} * BH_{t_0-t} \tag{4}$$

Among them, $BH_{t_0-t}{=}R_x(\alpha)_t * R_y(\beta)_t * R_z(\gamma)_t$

The conversion model[8][9], as shown in Fig.2, is established, and the actual laser direction is adjusted to the theoretical laser direction by rotation transformation. In the earth inertial coordinate system, the actual laser direction is SDn_t, and the theoretical laser direction is LDn_t. Setting X and Z axis as the axis with adjusting PTZ, the adjustment steps from the actual laser direction to the theoretical laser direction are as follows: firstly, the actual laser direction in the earth coordinate system is SDn_t, and the theoretical laser direction is LDn_t transform to the PTZ coordinate system $X_tY_tZ_t$:

In the PTZ coordinate system $X_tY_tZ_t$, the actual laser direction is SYn_t, and the theoretical laser direction is LYn_t:

$$SYn_t = SDn_t * BH_{D-Y_t} \tag{5}$$

$$LYn_t = LDn_t * BH_{D-Y_t} \tag{6}$$

BH_{D-Y_t} is the transformation matrix from the earth inertial coordinate system to the PTZ coordinate system at any t moment.

$$BH_{D-Y_t} = (YD_{t0} * BH_{t_0-t})^{-1} \tag{7}$$

The YD_{t0} is the azimuth matrix of the altimeter in the geodetic coordinate system at the start of calibration. $BH_{t_0-t}$is a rotation matrix from the PTZ coordinate system to the earth inertial coordinate system caused by the elevation angle change of the laser altimeter device from the t_0 moment to the arbitrary t time.

In the $X_tY_tZ_t$ PTZ coordinate system ,The actual laser direction $SYn_t$is located by adjusting the PTZ to rotate the $\delta_x$angle around the X axis to reach the position of the line 1 at the same height as the theoretical laser direction. And then the 1 line around the Z axis turn δ_z to the theory of laser direction LYn_t.

The position coordinates and pitch angles of the laser device in the inertial coordinate system D are unchanged,After adjustment.

The position coordinates of the laser altimeter are fixed in the PTZ coordinate system $X_tY_tZ_t$, and the pitch angle is: $(\alpha Y_t - \delta_X, \beta Y_t, \gamma Y_t - \delta_Z)$.

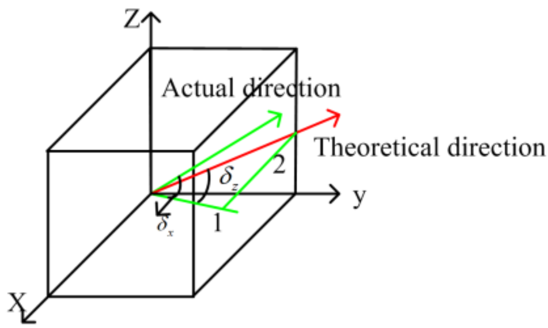

Fig. 2. transformation model from actual direction to theoretical direction

4. Calculation of coordinate system of adjusting quantity

4.1. Measurement initial time t_0

$SYn_0 = SDn_0 * BH_{D-Y_{t0}}$ can be known from 2.1, where $BH_{D-Y_{t0}}$ is the transformation matrix of t_0 from geodetic inertial coordinate system D to PTZ coordinate system at the moment.

$$BH_{D-Y_{t_0}} = BH^{-1}_{Y_{t_0}-D} = (\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha D_{t0}) & -\sin(\alpha D_{t0}) \\ 0 & \sin(\alpha D_{t0}) & \cos(\alpha D_{t0}) \end{bmatrix} * \begin{bmatrix} \cos(\beta D_{t0}) & 0 & \sin(\beta D_{t0}) \\ 0 & 1 & 0 \\ -\sin(\beta D_{t0}) & 0 & \cos(\beta D_{t0}) \end{bmatrix} * \begin{bmatrix} \cos(\gamma D_{t0}) & -\sin(\gamma D_{t0}) & 0 \\ \sin(\gamma D_{t0}) & \cos(\gamma D_{t0}) & 0 \\ 0 & 0 & 1 \end{bmatrix})^{-1} \quad (8)$$

So there are: $LYn_{t0} = SDn_{t0} * BH_{D-Y_{t_0}}$

4.2. Measure any t-1 moment

The azimuth elevation angle of the PTZ in the inertial coordinate system D is recorded as: $(\alpha D_{t-1}, \beta D_{t-1}, \gamma D_{t-1})$

Then the azimuth elevation angle of the two-axis adjustment PTZ head in the inertial coordinate system D is recorded as:

$$(\alpha D_{t-1}, \beta D_{t-1}, \gamma D_{t-1}) = (\alpha D_{t_0}, \beta D_{t_0}, \gamma D_{t_0}) * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha D_{t-1}) & -\sin(\alpha D_{t-1}) \\ 0 & \sin(\alpha D_{t-1}) & \cos(\alpha D_{t-1}) \end{bmatrix} * \begin{bmatrix} \cos(\gamma D_{t-1}) & -\sin(\gamma D_{t-1}) & 0 \\ \sin(\gamma D_{t-1}) & \cos(\gamma D_{t-1}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

In the geodetic inertial coordinate system D, the actual measuring direction is:

$$SDn_{t-1} = (xD_{t0}, yD_{t0}, zD_{t0}) * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha D_{t-1}) & -\sin(\alpha D_{t-1}) \\ 0 & \sin(\alpha D_{t-1}) & \cos(\alpha D_{t-1}) \end{bmatrix} * \begin{bmatrix} \cos(\gamma D_{t-1}) & -\sin(\gamma D_{t-1}) & 0 \\ \sin(\gamma D_{t-1}) & \cos(\gamma D_{t-1}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

So there are: $SYn_{t_{-1}} = SDn_{t-1} * BH_{D-Y_{t-1}}$, where $BH_{D-Y_{t-1}}$ is the transition matrix of inertial coordinate system to PTZ coordinate system at any $t-1$ moment.

$$BH_{D-Y_{t-1}} = (BH_{Y_{t-1}-D}) - 1 \tag{11}$$

Where

$$BH_{Y_{t-1}-D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha D_{t0}) & -\sin(\alpha D_{t0}) \\ 0 & \sin(\alpha D_{t0}) & \cos(\alpha D_{t0}) \end{bmatrix} * \begin{bmatrix} \cos(\beta D_{t0}) & 0 & \sin(\beta D_{t0}) \\ 0 & 1 & 0 \\ -\sin(\beta D_{t0}) & 0 & \cos(\beta D_{t0}) \end{bmatrix} * \begin{bmatrix} \cos(\gamma D_{t0}) & -\sin(\gamma D_{t0}) & 0 \\ \sin(\gamma D_{t0}) & \cos(\gamma D_{t0}) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha D_{t-1}) & -\sin(\alpha D_{t-1}) \\ 0 & \sin(\alpha D_{t-1}) & \cos(\alpha D_{t-1}) \end{bmatrix} * \begin{bmatrix} \cos(\gamma D_{t-1}) & -\sin(\gamma D_{t-1}) & 0 \\ \sin(\gamma D_{t-1}) & \cos(\gamma D_{t-1}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Therefore, at any time, after adjustment, the actual measurement direction SDn_{t-1} of the laser altimeter should be equal to the theoretical measurement direction LDn_{t-1}.

$$LDn_{t-1} = (xD_{t-1}, yD_{t-1}, zD_{t-1}) = SDn_{t-1} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\delta_x) & -\sin(\delta_x) \\ 0 & \sin(\delta_x) & \cos(\delta_x) \end{bmatrix} * \begin{bmatrix} \cos(\delta_z) & -\sin(\delta_z) & 0 \\ \sin(\delta_z) & \cos(\delta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

That is: $LDn_{t-1} = (xD_{t-1}, yD_{t-1}, zD_{t-1}) = (xD_{t0}, yD_{t0}, zD_{t0})*$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha D_{t-1}) & -\sin(\alpha D_{t-1}) \\ 0 & \sin(\alpha D_{t-1}) & \cos(\alpha D_{t-1}) \end{bmatrix} * \begin{bmatrix} \cos(\gamma D_{t-1}) & -\sin(\gamma D_{t-1}) & 0 \\ \sin(\gamma D_{t-1}) & \cos(\gamma D_{t-1}) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\delta_x) & -\sin(\delta_x) \\ 0 & \sin(\delta_x) & \cos(\delta_x) \end{bmatrix} * \begin{bmatrix} \cos(\delta_z) & -\sin(\delta_z) & 0 \\ \sin(\delta_z) & \cos(\delta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

By adjusting (10)(11)(12)(14), the coordinate system (δ_x δ_z) can be obtained as follows:

$$\delta_x = \arcsin \frac{Z_{Dt-1}}{\sqrt{A^2 + B^2}} - \varphi \tag{15}$$

Where : $A = -\{xD_{t_0} \sin(\gamma D_{t-1})[yD_{t0} \bullet \cos(\alpha D_{t-1}) + zD_{t0} \bullet \sin(\alpha D_{t-1})]\} \sin(\sigma_x)$

$$B = [-yD_{t0} \sin(\alpha D_{t-1}) + zD_{t0} \bullet \cos(\alpha D_{t-1})] \bullet \cos(\delta_x) \varphi = \arctan \frac{B}{A} \delta_z \tag{16}$$

Where $C = xD_{t0} \cos(\gamma D_{t-1}) + \sin(\gamma D_{t-1})[yD_{t0} \cos(\alpha D_{t-1}) + zD_{t0} \sin(\alpha D_{t-1})]$

$$D = \cos(\delta_x)\{-xD_{t0} \sin(\gamma D_{t-1}) + \cos(\gamma D_{t-1})[yD_{t0} \cos(\alpha D_{t-1}) + zD_{t0} \sin(\alpha D_{t-1})]\}$$

5. Simulation test verification

According to the expression of the amount of adjustment in the 3.2 section, The geocentric tracking algorithm is established through MatLab simulate software, and using the obtained adjustment expression into the model. As shown in Fig 3 and Fig 4, the measurement direction of the laser altimeter can accurately point to heart direction. The measurement error is about 5cm.

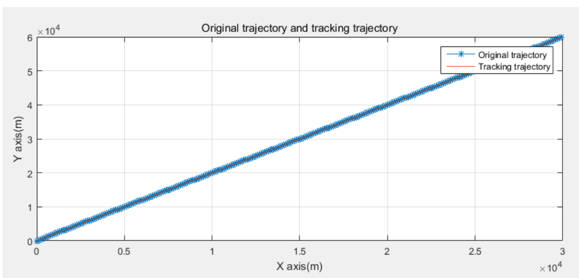

Fig. 3. tracking trajectory

Table 1. Interval measurement value

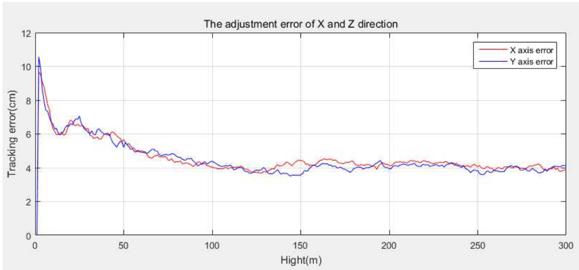

Fig. 4. tracking error

No.	Read the swing angle of the turntable	Read the X axis after adjusting the angle	Read the Z axis after adjusting the angle
1	25.0000°	0.0026°	0.0028°
2	20.0000°	0.0005°	-0.0016°
3	14.9999°	0.0015°	-0.0020°
4	10.0000°	0.0005°	0.0006°
5	5.0001°	0.0005°	-0.0014°
6	0.0001°	0.0000°	0.0004°
7	-5.0001°	0.0014°	0.0026°
8	-10.0001°	0.0006°	0.0028°
9	-15.0000°	0.0025°	0.0016°
10	-20.0000°	0.0015°	-0.0005°
11	-24.9999°	-0.0029°	-0.0013°

In order to reduce the random error in the actual calibration process, the adjustment error of the turntable is measured at different angles in the same position. Define 0° as geocentric direction. The motion trajectory of the two-dimensional tracking turntable is simulated with a platform of high precision and multi-degree of freedom. The response speed and tracking precision of the actuator are investigated. Turntable rotation angle to the United States PI's multi-degree of freedom platform as a benchmark (accuracy 0.0001°), each rotation of 5° for a measurement. Through the data obtained in Table 1, it can be seen that the adjustment error reaches 0.003°.

6. Summary

The thesis focuses on the analysis of direction directivity of laser altimeter measurement and established the position coordinates system. According to the actual measurement direction and the theoretical measurement direction of the laser altimeter at different measurement times, the modeling and analysis are made. Finally the mathematical expression of the amount of adjustment is derived. The tracking algorithm for the geocentric target is established by MatLab simulation. The simulation results prove that the tracking trajectory is basically consistent with the original trajectory, and the measurement error is 5cm. The reliability of the system is verified through the experimental platform, and the error demarcation indicates that the tracking precision can reach 0.003°. It meets the high performance requirements of laser altimeters.

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